

The equation of state of neutron star matter and the symmetry energy

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Abstract. We present an overview of microscopical calculations of the Equation of State (EOS) of neutron matter performed using Quantum Monte Carlo techniques. We focus to the role of the model of the three-neutron force in the high-density part of the EOS up to a few times the saturation density. We also discuss the interplay between the symmetry energy and the neutron star mass-radius relation.

The combination of theoretical models of the EOS with recent neutron stars observations permits us to constrain the value of the symmetry energy and its slope. We show that astrophysical observations are starting to provide important insights into the properties of neutron star matter.

1. Introduction

The knowledge of the Equation of State (EOS) of pure neutron matter is an important bridge between the symmetry energy, and neutron star properties. The symmetry energy E_s is the difference of nuclear matter and neutron matter energy, it gives the energy cost of the isospin-asymmetry in the homogeneous nucleonic matter. In the last few years the study of E_s has received considerable attention (see for example Ref. [1] for a recent experimental/theoretical review).

The role of the symmetry energy is essential to understand the mechanism of stability of very-neutron rich nuclei, but it is also related to many phenomena occurring in neutron stars. The stability of matter inside neutron stars is very sensitive to E_s and its first derivative. Around saturation density, neutrons tend to decay to protons through the β -decay, and the cooling of neutron stars is strongly connected to the proton/neutron ratio as a function of the density. This ratio is mainly governed by the behavior of E_s as a function of the density.

The inner crust of neutron stars, where the density is a fraction of nuclear densities, is mostly composed of neutrons surrounding a matter made of extremely-neutron rich nuclei that, depending on the density, may exhibit very different phases and properties. The extremely rich phase diagram of the neutron crustal matter is strongly related to the role of E_s . For example it governs the phase-transition between the crust and the core [2] and r -mode instability [3, 4].

The study of the EOS is particularly difficult because neutron matter is one of the most strongly-interacting fermionic systems. Neutron matter is often modeled by density functionals. Traditional Skyrme models (see for example Ref. [5] and references therein) and relativistic mean-field models (see for example Refs. [6, 7]) are two general classes of density functional theories.

Recently, a new methods based on microscopic nuclear Hamiltonians obtained from chiral effective field theories have been proposed. These nuclear forces are obtained following a systematic expansion in terms of momenta of the relevant degrees of freedom, and fit the nucleon-nucleon scattering data [8]. The nuclear Hamiltonian is then adjusted using renormalization group techniques to make the calculation perturbative [9]. New induced terms generated through the renormalization scheme have not yet been included though. These effects, as well as non-perturbative effects of three- and four-body forces, could be important [10].

The third class of these calculations uses nuclear potentials, like Argonne and Urbana/Illinois forces, that reproduces two-body scattering and properties of light nuclei with very high precision [11, 12]. In the latter case, the interaction has small non-local terms, giving the potentials a hard core. In this case calculations can be performed in a non-perturbative framework, and the strong correlations are solved by using correlated wave functions. In this paper we present results based on quantum Monte Carlo (QMC) methods. QMC methods have proven to be a very powerful tool to accurately study properties of light nuclei [13, 14] and nuclear matter [15] in a very similar way. They provide the unique technique to date to consistently study nuclear systems of different kind, both inhomogeneous and homogeneous matter, with the same accuracy and using the same Hamiltonians. Other techniques have been used to study nuclear and neutron matter and the symmetry energy based on Brueckner-Hartree-Fock theory (see for example Ref. [16] and references therein).

2. Nuclear Hamiltonian and Quantum Monte Carlo method

In our model, neutrons are non-relativistic point-like particles interacting via two- and three-body forces:

$$H = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}. \quad (1)$$

The two body-potential that we use is the Argonne AV8' [17], that is a simplified form of the Argonne AV18 [11]. Although simpler to use in QMC calculations, the AV8' provides almost the same accuracy as AV18 in fitting NN scattering data. The three-body force is not as well constrained as the NN interaction, but its inclusion in realistic nuclear Hamiltonians is important to correctly describe the binding energy of light nuclei [12]. Note that this is not necessary if phenomenological forces, like Skyrme or Gogny, or methods based on Relativistic Mean-Field theory are used.

The Urbana IX (UIX) three-body force has been originally proposed in combination with the Argonne AV18 and AV8' [18]. Although it slightly underbinds the energy of light nuclei, it has been extensively used to study the equation of state of nuclear and neutron matter [19, 20, 21]. The Illinois forces have been introduced to improve the description of both ground- and excited-states of light nuclei, showing an excellent accuracy [12, 14], but it produces an unphysical overbinding in pure neutron systems [22]. In this paper we shall present a study of the neutron matter EOS based on different models of three-neutron force giving specific values of the symmetry energy.

We solve the many-body ground-state using the Auxiliary Field Diffusion Monte Carlo (AFDMC) originally introduced by Schmidt and Fantoni [23]. The main idea of QMC methods is to evolve a many-body wave function in imaginary-time:

$$\Psi(\tau) = \exp[-H\tau]\Psi_v, \quad (2)$$

where Ψ_v is a variational ansatz, and H is the Hamiltonian of the system. In the limit of $\tau \rightarrow \infty$, Ψ approaches the ground-state of H . The evolution in imaginary-time is performed by sampling

configurations of the system using Monte Carlo techniques, and expectation values are evaluated over the sampled configurations. For more details see for example Refs. [13, 20].

The Green's Function Monte Carlo (GFMC) provided to be extremely accurate to study properties of light nuclei. The variational wave function includes all the possible spin/isospin states of nucleons and it provides a good variational ansatz to start the projection in the imaginary-time. The exponential growing of this states limits the calculation to the ^{12}C [24]. The AFDMC method does not explicitly include all the spin/isospin states in the wave function, but they are instead sampled using the Hubbard-Stratonovich transformation. The calculation can be then extended up to many neutrons, making the simulation of homogeneous matter possible. The AFDMC has proven to be very accurate when compared to GFMC calculation of energies of neutrons confined in an external potential [25].

3. Symmetry energy

The symmetry energy is defined as the difference between pure neutron matter and symmetric nuclear matter. The energy of nuclear matter is often expressed as an expansion in even powers of the isospin-asymmetry

$$E(\rho, x) = E_0(\rho) + E_s^{(2)}(\rho)(1 - 2x)^2 + E_s^{(4)}(\rho)(1 - 2x)^4 + \dots, \quad (3)$$

where E is the energy per particle, $x = \rho_p/(\rho_p + \rho_n)$ is the proton fraction, ρ is the density of the system, $E_s^{(2n)}$ are coefficients multiplying the isospin asymmetry terms $(1 - 2x)^{2n}$, and $E_0(\rho) = E(\rho, x = 0.5)$ is the energy of symmetric nuclear matter. The symmetry energy E_s is given by

$$E_s(\rho) = E(\rho, 0) - E_0(\rho). \quad (4)$$

The energy at saturation of symmetric nuclear matter extrapolated from the binding energy of heavy nuclei is $E(\rho_0) = -16$ MeV, where $\rho_0 = 0.16 \text{ fm}^{-3}$ is the saturation density. The symmetry energy around saturation ρ_0 can be expanded as

$$E_s(\rho) \Big|_{\rho_0} = E_{sym} + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \dots, \quad (5)$$

where L is related to the slope of E_s . By combining the above equations, we can easily relate the symmetry energy to the EOS of pure neutron matter at density close to ρ_0 .

4. Results

In this section we present QMC results for pure neutron matter. There are several reasons to focus on pure neutron matter. First, the three-body interaction is non-zero only in the $T = 3/2$ isospin-channel (T is the total isospin of three-nucleons), while in the presence of protons there are also contributions in $T = 1/2$. The latter term is the dominant one in nuclei, and only weakly accessible by studying properties of nuclei. Second, the EOS of pure neutron matter is closely related to the structure of neutron stars.

We present several EOS obtained using different models of three-neutron force in Fig. 1. The two solid lines correspond to the EOS calculated using the NN potential alone and including the UIX three-body force [18]. The effect of using different models of three-neutron force is clear in the two bands, where the high density behavior is showed up to about $3\rho_0$. At such high density, the various models giving the same symmetry energy at saturation produce an uncertainty to the EOS of about 20 MeV. The EOS obtained using QMC can be conveniently fit using the following functional [20]:

$$E(\rho) = a \left(\frac{\rho}{\rho_0} \right)^\alpha + b \left(\frac{\rho}{\rho_0} \right)^\beta, \quad (6)$$

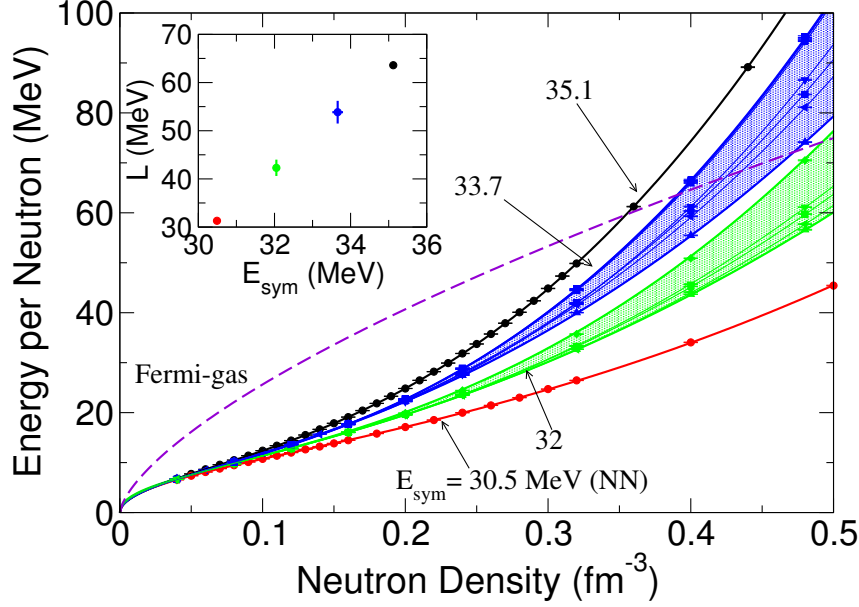


Figure 1. The QMC equation of state of neutron matter for various Hamiltonians. The red (lower) curve is obtained by including the NN (Argonne AV8') alone in the calculation, and the black one is obtained by adding the Urbana IX three-body force. The green and blue bands correspond to EOS giving the same E_{sym} (32 and 33.7 MeV respectively), and are obtained by using several models of three-neutron force. In the inset we show the value of L as a function of E_{sym} obtained by fitting the EOS. The figure is taken from Ref. [21].

where E is the energy per neutron, $\rho_0 = 0.16 \text{ fm}^{-3}$, and a , b , α and β are free parameters. The parametrizations of the EOS obtained from different nuclear Hamiltonians is given in Ref. [21].

At ρ_0 symmetric nuclear matter saturates, and we can extract the value of E_{sym} and L directly from the pure neutron matter EOS. The result of fitting Eq. 5 to the pure neutron matter EOS is shown in the inset of Fig. 1. The error bars are obtained by taking the maximum and minimum value of L for a given E_{sym} , and the curves obtained with NN and NN+UIX are thus without error bars. From the plot it is clear that within the models we consider, the correlation between L and E_{sym} is linear and quite strong.

When the EOS of the neutron matter has been specified, the structure of an idealized spherically-symmetric neutron star model can be calculated by integrating the Tolman-Oppenheimer-Volkoff (TOV) equations:

$$\frac{dP}{dr} = -\frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]}, \quad (7)$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho, \quad (8)$$

where $P = \rho^2(\partial E/\partial \rho)$ and $\epsilon = \rho(E + m_N)$ are the pressure and the energy density, m_N is the neutron mass, $m(r)$ is the gravitational mass enclosed within a radius r , and G is the gravitational constant. The solution of the TOV equations for a given central density gives the profiles of ρ , ϵ and P as functions of radius r , and also the total radius R and mass $M = m(R)$. The total radius R is given by the condition $P(R) = 0$.

The mass of a neutron star as a function of its radius is shown in Fig. 2. The two bands correspond to the result obtained using the two sets of EOS giving the same value of E_{sym}

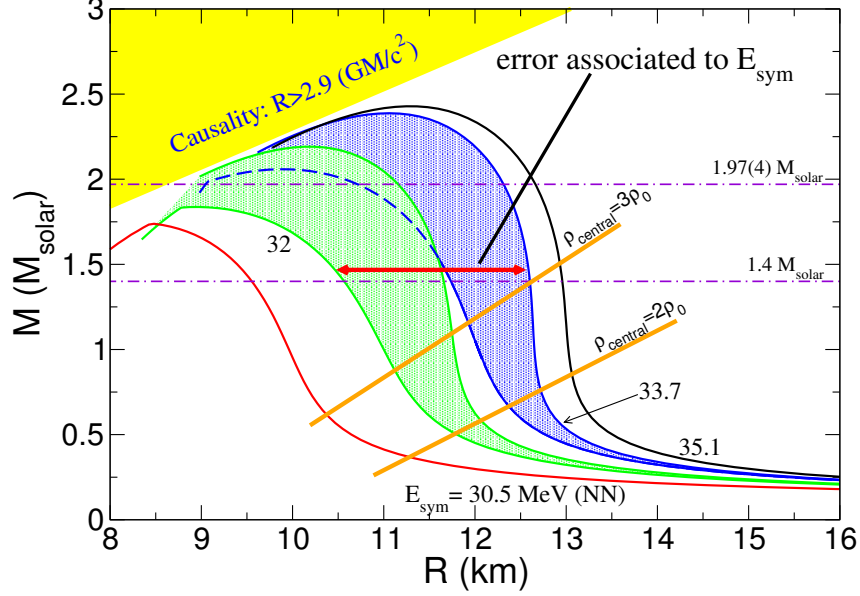


Figure 2. The mass-radius relation of neutron stars obtained from the EOS calculated using QMC. The various colors represent the $M - R$ result obtained from the corresponding EOS obtained as described in Fig. 1. The two horizontal lines show the value of $M = 1.4$ and $1.97(4)M_{\text{solar}}$ [26]. The figure is taken from Ref. [21].

indicated in the figure. As in the case of the EOS, it is clear that the main source of uncertainty to the radius of a neutron star with $M = 1.4M_{\text{solar}}$ is due to the uncertainty of E_{sym} rather than the model of the three-neutron force. It has to be noted that we have used the EOS of pure neutron matter without imposing the β -equilibrium, so in our model we don't have protons. However the addition of a small proton fraction would change the radius R only slightly [15, 19] smaller than other uncertainties in the EOS that we have discussed.

The EOS of neutron matter and its properties can also be extracted from astrophysical observations [27]. By combining the Bayesian analysis with the model of neutron matter of Eq. 6 it is possible to compare the QMC prediction with observations [28], and to extract E_{sym} and L :

$$E_{\text{sym}} = a + b + 16, \quad L = 3(a\alpha + b\beta), \quad (9)$$

and from neutron stars we obtain the constraints $31.2 < E_{\text{sym}} < 34.3$ MeV and $36.6 < L < 55.1$ MeV [28] (at the 2- σ confidence level) in agreement with the QMC predictions.

In order to better appreciate the agreement between theoretical calculation with the neutron star structure obtained from observations, we show a comparison in Fig. 3. In the figure the two green and blue bands correspond to the $M - R$ relation obtained from the EOS of Fig. 1, and the black and red bands represent the astrophysical observation of Ref. [28] using different models for the high-density EOS.

5. Conclusions

We have presented a theoretical calculation of the neutron matter EOS using QMC methods. This technique permits to study the ground-state of strongly interacting Fermi systems in a full non-perturbative way. Calculations have been performed using a modern nucleon-nucleon interaction that fit the phase shifts with high accuracy. We have studied the effect of using

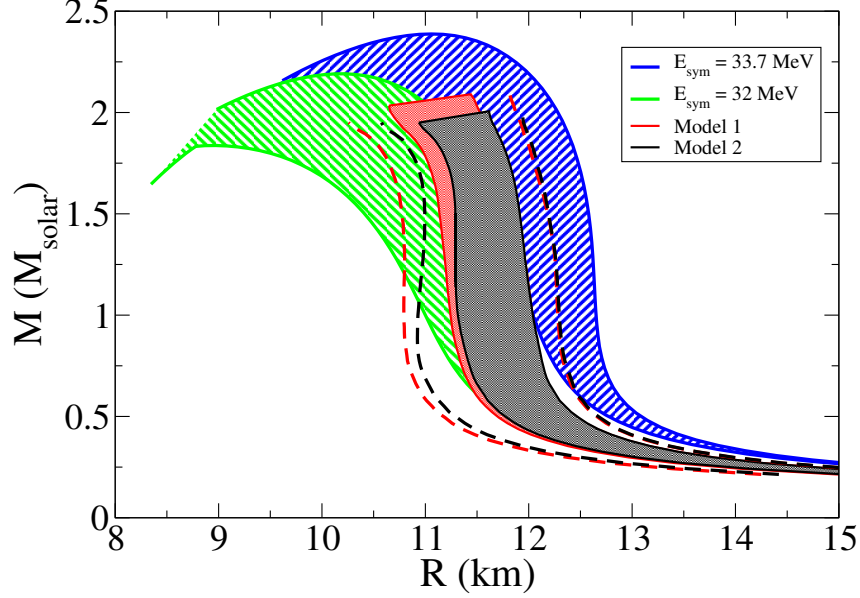


Figure 3. The comparison of $M - R$ relation of neutron stars obtained from QMC calculations and observations. The blue and green bands are the same of Fig. 2, and correspond to EOS giving the value of E_{sym} indicated in the legend. The black and red bands are obtained from neutron star observations of Ref. [28] at the $1-\sigma$ confidence level (dashed lines at $2-\sigma$), and they correspond to different models of the high-density part.

different microscopical models of three-neutron forces, by quantifying their role in the high-density EOS up to $3\rho_0$. By performing simulations using Hamiltonians that give different values of the symmetry energy we conclude that, at present, the uncertainty to the EOS is mainly due to the poor constrain of E_{sym} rather than the model of the three-neutron force. From our calculation we have extracted the relation between L and E_{sym} suggesting that they are quite strongly linearly related.

We also provide new constraints from astrophysical observations. By combining the recent analysis of Steiner *et al.* with an empirical EOS which form is suggested by QMC simulations, we provided a new constrain to the value of the symmetry energy and its slope at saturation [28]. The result is compatible with several experimental measurements [1]. We find good agreement for the $M - R$ relation of neutron stars given by QMC prediction and from observations.

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